# FULLY BAYESIAN FIELD SLAM USING GAUSSIAN MARKOV RANDOM FIELDS

Huan N. Do, Mahdi Jadaliha, Mehmet Temel, and Jongeun Choi

## ABSTRACT

This paper presents a fully Bayesian way to solve the simultaneous localization and spatial prediction problem using a Gaussian Markov random field (GMRF) model. The objective is to simultaneously localize robotic sensors and predict a spatial field of interest using sequentially collected noisy observations by robotic sensors. The set of observations consists of the observed noisy positions of robotic sensing vehicles and noisy measurements of a spatial field. To be flexible, the spatial field of interest is modeled by a GMRF with uncertain hyperparameters. We derive an approximate Bayesian solution to the problem of computing the predictive inferences of the GMRF and the localization, taking into account observations, uncertain hyperparameters, measurement noise, kinematics of robotic sensors, and uncertain localization. The effectiveness of the proposed algorithm is illustrated by simulation results as well as by experiment results. The experiment results successfully show the flexibility and adaptability of our fully Bayesian approach in a data-driven fashion.

*Key Words:* Vision-based localization, spatial modeling, simultaneous localization and mapping (SLAM), Gaussian process regression, Gaussian Markov random field.

### I. INTRODUCTION

Simultaneous localization and mapping (SLAM) addresses the problem of a robot exploring an unknown environment under localization uncertainty [1]. The SLAM technology is essential to robotic tasks [2]. The variations of the SLAM problem are surveyed and categorized with different perspectives in [3]. In general, most SLAM problems have geometric models [1,3,4]. For example, a robot learns the locations of the landmarks while localizing itself using triangulation algorithms. Such geometric models could be classified in two groups, namely, a sparse set of features which can be individually identified, often used in Kalman filtering methods [1], and a dense representation such as an occupancy grid, often used in particle filtering methods [5].

However, there are a number of inapplicable situations. For example, underwater autonomous gliders for ocean sampling cannot find usual geometrical models from measurements of environmental variables such as pH, salinity, and temperature [6]. Therefore, in contrast to popular geometrical models, there is a growing number of practical scenarios in which measurable spatial fields are exploited instead of geometric models. In this regard, we may consider localization using various spatially distributed (continuous) signals such as distributed wireless ethernet signal strength [7] or multi-dimensional magnetic fields [8]. For instance, a localization approach (so-called vector field SLAM) that learns the spatial variation of an observed continuous signal was developed by modeling the signal as a piecewise linear function and applying SLAM subsequently [9]. Gutmann et al. [9] demonstrated the approach by an indoor experiment in which a mobile robot performed localization based on an optical sensor detecting unique infrared patterns projected on the ceiling. Do et al. [10] localized a mobile robot for both indoors and outdoors by modeling the high-dimensional visual feature vector extracted from omni-directional images as a multivariate Gaussian random field.

In this paper, we consider scenarios without geometric models and tackle the problem of simultaneous localization and prediction of a spatial field of interest. With an emphasis on spatial field modeling, our problem will be referred to as a field SLAM for the rest of the paper.

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Spatial modeling and prediction techniques for random fields have been exploited for mobile robotic sensors [6,11–18]. Random fields such as Gaussian processes and Gaussian Markov random fields (GMRFs) [19,20] have been frequently used for mobile sensor networks to statistically model physical phenomena such as harmful algal blooms, pH, salinity, temperature, and wireless signal strength, [15–18,21].

The recent research efforts that are closely related to our problem are summarized as follows. The authors in [22] formulated Gaussian process regression under uncertain localization, distributed versions of which are also reported in [23]. In [24], a physical spatio-temporal random field was modeled as a GMRF with uncertain hyperparameters and the prediction problems with localization uncertainty were tackled. However, kinematics or dynamics of the sensor vehicles were not incorporated in [22,24]. Brooks et al. [25] used Gaussian process regression to model geo-referenced sensor measurements (obtained from a camera). The noisy measurements and their exact sampling positions were utilized in the training step. Then, the locations of newly sampled measurements were estimated by a maximum likelihood. However, this was not a SLAM problem since the training step had to be performed a priori for a given environment [25]. The authors in [8,26] used the Gaussian process regression to implement SLAM based on a magnetic field and experimentally showed its feasibility. O'Callaghan [27] used laser range-finder data to probabilistically classify a robot's environment into a region of occupancy. They provided a continuous representation of a robot's surroundings by employing a Gaussian process. In [7], a WiFi SLAM problem was solved using a Gaussian process latent variable model (GP-LVM).

To the best of our knowledge, the research efforts on the field SLAM problem up to date have estimated hyperparameters off-line *a priori*. Therefore, they have not addressed the uncertainties in the hyperparameters of the spatial model (e.g., Gaussian process) in a fully Bayesian manner. For example, Gutmann *et al.* [9] assumed a piecewise linear function for the field and estimated the linear model off-line *a priori*. Additionally Do *et al.* [10] performed Gaussian process regression by estimating its hyperparameters off-line to build the localization model *a priori*.

The main contribution of our work is to incorporate the uncertainties in the hyperparameters into a model such that they converge to the optimal values in a data-driven fashion. The advantage of our fully Bayesian approach in this paper is well demonstrated by simulation (Section III) and experiment results (Section IV). The results show flexibility and adaptability of our fully Bayesian approach in a data-driven fashion.

In this paper, we first formulate the field SLAM problem in order to simultaneously localize robotic sensors and predict a spatial random field of interest using sequentially collected noisy observations by robotic sensors. The set of observations consists of observed noisy positions of robotic sensing vehicles and noisy measurements of a spatial field. To be flexible, the spatial field of interest is modeled by a GMRF with uncertain hyperparameters. We then derive an approximate Bayesian solution to the problem of computing the predictive inferences of the GMRF and the localization. Our method takes into account observations, uncertain hyperparameters, measurement noise, kinematics of robotic sensing vehicles, and noisy localization. The effectiveness of the proposed algorithm is illustrated by both simulation and experiment results. In particular, the experiment with a mobile robot equipped with a panoramic vision camera shows that the fully Bayesian approach successfully converges to the maximum likelihood estimation of the hyperparameter vector among a number of prior candidates.

A preliminary version of this paper without experimental validation was reported at the 2013 American Control Conference [28].

**Notation.** Standard notations will be used throughout the paper. Let  $\mathbb{R}$  and  $\mathbb{Z}_{>0}$  be the sets of real and positive integer numbers, respectively. The collection of *n m*-dimensional vectors  $\{q_i \in \mathbb{R}^m | i = 1, \dots, n\}$  is denoted by  $q := \operatorname{col}(q_1, \dots, q_n) \in \mathbb{R}^{nm}$ . The operators of expectation and covariance matrix are denoted by  $\mathbb{E}$  and Cov, respectively. A random vector *x*, which has a multivariate normal distribution of mean vector  $\mu$  and covariance matrix  $\Sigma$ , is denoted by  $x \sim \mathcal{N}(\mu, \Sigma)$ . For given  $G = \{c, d\}$ and  $H = \{1, 2\}$ , the multiplication between two sets is defined as  $H \times G = \{(1, c), (1, d), (2, c), (2, d)\}$ . Other notations will be explained in due course.

# II. SEQUENTIAL BAYESIAN INFERENCE FOR FIELD SLAM

In this section, we provide a foundation of our approach by formulating the field SLAM problem and subsequently by providing its approximate sequential Bayesian solution.

# 2.1 From Gaussian processes to Gaussian Markov random fields

In this section, we introduce a GMRF as a discretized Gaussian process on a lattice. Consider a Gaussian process

$$z(q) \sim \mathcal{GP}(m(q), \mathcal{K}(q, q')),$$

where m(q) is a mean function,  $\mathcal{K}(q, q')$  is a covariance function defined with respect to locations q, q' in a compact domain, *i.e.*,  $q, q' \in S_c := [0 \ x_{max}] \times [0 \ y_{max}]$ , and z(q) is the realization of the Gaussian process z at the robot position q. We discretize the compact domain  $S_c$  into *n* spatial sites  $S := \{s^{[1]}, \dots, s^{[n]}\} \subset \mathbb{R}^d$ , where  $n = hx_{max} \times hy_{max}$ . h is chosen such that  $n \in \mathbb{Z}$ . Note that  $n \to \infty$  as  $h \to \infty$ . The collection of realized values of the random field in S is denoted by  $z := (z^{[1]}, \cdots, z^{[n]})^T \in$  $\mathbb{R}^n$ , where  $z^{[i]} := z(s^{[i]})$ . The prior distribution of z is then given by  $z \sim \mathcal{N}(\mu, \Sigma)$ , and so we have

$$\pi(z) \propto \exp\left(-\frac{1}{2} \left(z-\mu\right)^T \Sigma^{-1} \left(z-\mu\right)\right),\,$$

where  $\mu := (m(s^{[1]}), \cdots, m(s^{[n]}))^T \in \mathbb{R}^n$  and the *i*, *j*-th element of  $\Sigma$  is defined as  $\Sigma^{[ij]} := \mathcal{K}(s^{[i]}, s^{[j]}) \in \mathbb{R}$ . The prior distribution of z can be written by a precision matrix  $Q = \Sigma^{-1}$ , *i.e.*,  $z \sim \mathcal{N}(\mu, Q^{-1})$ . This can be viewed as a discretized version of the Gaussian process (or a GMRF) with a precision matrix Q on S. Note that Q of this GMRF is not sparse. However, a sparse version of O, *i.e.*,  $\hat{Q}$  with local neighborhood that can represent the original Gaussian process can be found, for example, making  $\hat{Q}$  close to Q in some norm [29–31]. This approximate GMRF will be computationally efficient due to the sparsity of  $\hat{Q}$ . In our approach, any model for  $\mu_{\theta}$  and  $Q_{\theta}$ , where  $\theta$  is the hyperparameter vector, can be used. In our simulation and experimental studies, we will use a GMRF with a sparse precision matrix that represents a Gaussian process precisely as shown in [24,32].

#### 2.2 Multiple robotic sensors

Consider N spatially distributed robots with sensors indexed by  $j \in \mathcal{J} := \{1, \dots, N\}$  sampling at time  $t \in$  $\mathbb{Z}_{>0}$ . Suppose that the sampling time  $t \in \mathbb{Z}$  is discrete. Recall that the surveillance region is now discretized as a lattice that consists of *n* spatial sites, the set of which is denoted by S. Let n spatial sites in S be indexed by  $\mathcal{I} := \{1, \dots, n\}, \text{ and } z := \operatorname{col} (z^{[1]}, \dots, z^{[n]}) \in \mathbb{R}^n \text{ be the}$ corresponding static values of the scalar field at n special sites. We denote all robots' locations at time t by  $q_t :=$  $\operatorname{col}(q_t^{[1]}, \cdots, q_t^{[N]}) \in S^N$ , the observations made by all robots at time t by  $\tilde{z}_t := \operatorname{col}\left(\tilde{z}_t^{[1]}, \cdots, \tilde{z}_t^{[N]}\right) \in \mathbb{R}^N$ , and the observed positions of all robots at time t by  $\tilde{q}_t :=$  $\operatorname{col}\left(\tilde{q}_{t}^{[1]}, \cdots, \tilde{q}_{t}^{[N]}\right) \in S^{N}$ .  $\tilde{q}_{t}$  and  $\tilde{z}_{t}$  are noisy observations of  $q_t$  and z at time t, respectively.

At time t, robot j samples a noise corrupted measurement at its current location  $q_i^{[j]} = s^{[i]} \in S, \forall j \in$  $\mathcal{J}, i \in \mathcal{I}, i.e.,$ 

$$\tilde{z}_{t}^{[j]} = z^{[i]} + \epsilon_{t}^{[j]}, \tag{1}$$

where the measurement errors  $\{\epsilon_t^{[j]}\}$  are assumed to be the independent and identically distributed (i.i.d.) Gaussian white noise, *i.e.*,  $\epsilon_t^{[j]} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_{\epsilon}^2)$ . The measurement noise level  $\sigma_{\epsilon}^2 > 0$  is assumed to be known, and we define  $\epsilon_t := \operatorname{col}\left(\epsilon_t^{[1]}, \cdots, \epsilon_t^{[N]}\right) \in \mathbb{R}^N.$ 

In addition, at time t, robot j samples a noisy observation of its own vehicle position.

$$\tilde{q}_t^{[j]} = q_t^{[j]} + e_t^{[j]}, \tag{2}$$

where the observation errors  $\{e_t^{[j]}\}$  are distributed by

 $e_t^{[j]} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_e^2 I).$ The observation noise level  $\sigma_e^2 > 0$  is assumed to be known, and we define  $e_t := \operatorname{col}\left(e_t^{[1]}, \cdots, e_t^{[N]}\right) \in \mathbb{R}^{d \times N}$ .

We then have the following collective notations.

$$\begin{aligned} \tilde{z}_t &= H_{q_t} z + \epsilon_t, \\ \tilde{q}_t &= L_t q_t + \epsilon_t, \end{aligned} \tag{3}$$

where  $L_t$  is the observation matrix for the vehicle states, and  $H_{q_t} \in \mathbb{R}^{N \times n}$  is defined by

$$H_{q_t}^{[ij]} = \begin{cases} 1, \text{ if } s^{[j]} = q_t^{[i]}, \\ 0, \text{ otherwise.} \end{cases}$$

#### 2.3 Kinematics of robotic vehicles

In this section, we introduce a specific model for the motion of robotic vehicles for a clear presentation. Each robotic sensor is modeled by a nonholonomic differentially driven vehicle in a two dimensional domain, *i.e.*,  $S \in \mathbb{R}^2$ . In this case, the kinematics of robot *i* can be given by

$$\begin{bmatrix} \dot{q}_{l}^{[1,i]} \\ \dot{q}_{l}^{[2,i]} \end{bmatrix} = \begin{bmatrix} u_{l}^{[i]} \cos \psi_{l}^{[i]} \\ u_{l}^{[i]} \sin \psi_{l}^{[i]} \end{bmatrix} + w_{l}^{[i]},$$
(4)

where  $\{q_t^{[1,i]}, q_t^{[2,i]}\}, \{\psi_t^{[i]}\}, \{u_t^{[i]}\}, and w_t^{[i]}$  denote the inertial position, the orientation, the linear speed, and the system noise of robot *i* in time *t*, respectively.

We may assume that the orientation  $\{\psi_t^{[i]} | \forall i \in \mathcal{J}\}$ can be updated from the turn rate  $\{\eta_t^{[i]} | \forall i \in \mathcal{J}\}$ . In this case, the kinematics of the vehicle network can be further described in a collective and discretized form as follows.

$$q_{t+1} = q_t + F_t u_t + w_t, (5)$$

where  $u_t$  is the known control input and  $w_t$  in the i.i.d. white noise realized by a known normal distribution  $\mathcal{N}(0, \Sigma_{w_t})$ .

# 2.4 Problem formulation and its Bayesian predictive inference

In this section, we formulate the field SLAM problem with a GMRF over a lattice and provide its Bayesian solution. To be precise, we present the following assumptions A1-A5 for the problem formulation.

- A1. The scalar random field z is generated by a GMRF model which is given by  $z \sim \mathcal{N}(\mu_{\theta}, Q_{\theta}^{-1})$ , where  $\mu_{\theta}$  and  $Q_{\theta}$  are given functions of a hyperparameter vector  $\theta$ .
- **A2.** The noisy measurements  $\{\tilde{z}_t\}$  and the noisy sampling positions  $\{\tilde{q}_t\}$ , as in (3), are collected by robotic sensors in time  $t = 1, 2, \cdots$ .
- A3. The control inputs  $u_t$  and the orientation  $\psi_t$  are known deterministic values at time t.
- A4. The prior distribution of the hyperparameter vector  $\theta$  is discrete with a support  $\Theta = \{\theta^{(1)}, \dots, \theta^{(L)}\}.$
- A5. The prior distribution of the sampling positions in time t,  $\pi(q_t)$ , is discrete with a support  $\Omega(t) = \{q_t^{(k)} | k \in \mathcal{L}(t)\}$ , which is given at time t. Here,  $\mathcal{L}(t) = \{1, \dots, v(t)\}$  denotes the index in the support and v(t) is the number of the probable possibilities for  $q_t$ .

**Remark II.1.** A1 states that the spatial field is modeled by any GMRF. A2 is a standard noise assumption. Regarding A3, when  $\psi_t$  is not accurately integrated from noisy turn rate  $\eta_t$ , we will show how to relax this condition of accurate orientation using the extended Kalman filter (EKF) [33]. A4 and A5 indicate that we consider the hyperparameter vector and sampling positions as discrete random vectors.

**Problem II.2.** Consider the assumptions A1-A5. Our field SLAM problem is to find the predictive distribution, mean, and variance of *z* conditional on  $D_t := \{\tilde{z}_{1:t}, \tilde{q}_{1:t}\}$ , where

$$\tilde{z}_{1:t} := \operatorname{col}(\tilde{z}_1, \cdots, \tilde{z}_t) \in \mathbb{R}^{N_t}, \\ \tilde{q}_{1:t} := \operatorname{col}(\tilde{q}_1, \cdots, \tilde{q}_t) \in \mathcal{S}^{N_t}.$$

The solution to Problem II.2 is derived as follows. The distribution of the GMRF is given by  $\pi (z|\theta, D_{t-1}) = \mathcal{N} (\mu_{z|\theta,D_{t-1}}, \Sigma_{z|\theta,D_{t-1}})$ . Recall that the evolution of  $q_t$  is given by (5) and the input  $u_t$  is a known deterministic value at time *t*. Therefore,  $\pi (q_t | D_{t-1})$  can be updated by the Gaussian approximation of  $\pi (q_{t-1|D_{t-1}})$ .

$$\pi \left( q_{t} | \mathcal{D}_{t-1} \right) \approx \\ \mathcal{N} \left( \mu_{q_{t-1} | \mathcal{D}_{t-1}} + F_{t-1} u_{t-1}, \Sigma_{q_{t-1} | \mathcal{D}_{t-1}} + \Sigma_{w_{t-1}} \right).$$

Similarly,  $\pi \left(\tilde{z}_{t} | \theta, \mathcal{D}_{t-1}, q_{t}\right)$  is updated by the Gaussian approximation of  $\pi(z|\theta, \mathcal{D}_{t-1})$  as follows.

$$\pi \left( \tilde{z}_{t} | \theta, \mathcal{D}_{t-1}, q_{t} \right) \approx \mathcal{N} \left( H_{q_{t}} \mu_{z | \theta, \mathcal{D}_{t-1}}, \Sigma_{\epsilon_{t}} + H_{q_{t}} \Sigma_{z | \theta, \mathcal{D}_{t-1}} H_{q_{t}}^{T} \right).$$
(6)

**Remark II.3.** For the sake of decreasing complexity and to make the entire algorithm sequential, the distribution of  $q_t | D_{t-1}$  and  $\tilde{z}_t | \theta, D_{t-1}, qt$  are approximated by normal distributions.

The joint distribution  $z, q_t, \theta | D_{t-1}$  is obtained as follows.

$$\begin{split} \pi \left( z, q_t, \theta | \mathcal{D}_{t-1} \right) &= \\ \pi \left( z | \theta, q_t, \mathcal{D}_{t-1} \right) \pi \left( \theta | q_t, \mathcal{D}_{t-1} \right) \pi \left( q_t | \mathcal{D}_{t-1} \right). \end{split}$$

Note that  $\tilde{q}_t$  and  $\tilde{z}_t$  are conditionally independent with respect to  $\{z, \theta, D_{t-1}\}$  and  $\tilde{q}_t$ , respectively. We can then simplify  $\pi(\tilde{q}_t | z, \theta, q_t, D_{t-1})$  and  $\pi(\tilde{z}_t | z, \theta, q_t, \tilde{q}_t, D_{t-1})$  by  $\pi(\tilde{q}_t | q_t)$  and  $\pi(\tilde{z}_t | z, \theta, q_t, D_{t-1})$ , respectively. The observation model is given by (3), thus the probabilities of the observed data are  $\pi(\tilde{z}_t | z, q_t) = \mathcal{N}(H_{q_t} z, \Sigma_{\epsilon_t})$  and  $\pi(\tilde{q}_t | q_t) = \mathcal{N}(L_t q_t, \Sigma_{\epsilon_t})$ . The measured random variables have the following conditional joint distribution,

$$\pi\left(\tilde{z}_{t},\tilde{q}_{t}|z,q_{t},\theta,\mathcal{D}_{t-1}\right)=\pi\left(\tilde{z}_{t}|z,\theta,q_{t},\mathcal{D}_{t-1}\right)\pi\left(\tilde{q}_{t}|q_{t}\right).$$

From Bayes' rule, the posterior joint distribution of the scalar field values, the sampling positions, and the hyperparameter vector is given as follows.

$$\pi\left(z, q_{t}, \theta | \mathcal{D}_{t}\right) = \frac{\pi\left(\tilde{z}_{t}, \tilde{q}_{t} | z, q_{t}, \theta, \mathcal{D}_{t-1}\right) \pi\left(z, qt, \theta | \mathcal{D}_{t-1}\right)}{\pi\left(\tilde{z}_{t}, \tilde{q}_{t} | \mathcal{D}_{t-1}\right)}$$

In addition,  $\pi(q_t, \theta | D_t) = \int \pi(z, q_t, \theta | D_t) dz$  is given as follows.

$$\int \pi \left( z, q_{t}, \theta | \mathcal{D}_{t} \right) dz$$

$$= \frac{\pi \left( \tilde{q}_{t} | q_{t} \right) \pi \left( \theta | q_{t}, \mathcal{D}_{t-1} \right) \pi \left( q_{t} | \mathcal{D}_{t-1} \right)}{\pi \left( \tilde{z}_{t}, \tilde{q}_{t} | \mathcal{D}_{t-1} \right)}$$

$$\times \int \pi \left( z | \theta, q_{t}, \mathcal{D}_{t-1} \right) \pi \left( \tilde{z}_{t} | z, \theta, q_{t}, \mathcal{D}_{t-1} \right) dz,$$
(7)

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where  $\int \pi (z | \theta, q_t, \mathcal{D}_{t-1}) \pi (\tilde{z}_t | z, \theta, q_t, \mathcal{D}_{t-1}) dz = \pi (\tilde{z}_t | q_t, \theta, \mathcal{D}_{t-1})$ , and  $\pi (\tilde{z}_t | q_t, \theta, \mathcal{D}_{t-1})$  is given by (6).

**Remark II.4.** From the Bayes' rule,  $\pi(\theta|q_t, D_{t-1})$  is given by  $\frac{\pi(\theta,q_t|D_{t-1})}{\pi(q_t|D_{t-1})}$ . We can compute  $\pi(\theta, q_t|D_{t-1})$  for all the possible combinations of  $q_t$  in the previous iteration using (7). However, for the sake of reducing the computational cost, we approximate  $\pi(\theta|q_t, D_{t-1})$  by  $\pi(\theta|D_{t-1})$ . Therefore, we have

$$\frac{\pi\left(q_{t},\theta|\mathcal{D}_{t}\right)\approx}{\frac{\pi\left(\tilde{q}_{t}|q_{t}\right)\pi\left(\theta|\mathcal{D}_{t-1}\right)\pi\left(q_{t}|\mathcal{D}_{t-1}\right)\pi\left(\tilde{z}_{t}|q_{t},\theta,\mathcal{D}_{t-1}\right)}{\pi\left(\tilde{z}_{t},\tilde{q}_{t}|\mathcal{D}_{t-1}\right)}.$$

Marginalizing out uncertainties on the possible  $q_t$ and  $\theta$ , we obtain the following.

$$\pi \left( z, \theta | \mathcal{D}_t \right) = \sum_{q_t \in \Omega(t)} \pi \left( z, q_t, \theta | \mathcal{D}_t \right),$$
$$\pi \left( z | \mathcal{D}_t \right) = \sum_{\theta \in \Theta} \pi \left( z, \theta | \mathcal{D}_t \right).$$

Our estimation of  $q_t$  and  $\theta$  can be updated using measured data up to time t as follows.

$$\pi \left( q_t | \mathcal{D}_t \right) = \sum_{\theta \in \Theta} \pi \left( q_t, \theta | \mathcal{D}_t \right),$$
  
$$\pi \left( \theta | \mathcal{D}_t \right) = \sum_{q_t \in \Omega(t)} \pi \left( q_t, \theta | \mathcal{D}_t \right).$$
(8)

The predictive probability and mean of  $z|\theta, D_t$  are obtained as follows.

$$\pi \left( z | \theta, \mathcal{D}_t \right) = \frac{\pi \left( z, \theta | \mathcal{D}_t \right)}{\pi \left( \theta | \mathcal{D}_t \right)},$$
$$\mu_{z | \theta, \mathcal{D}_t} = \frac{1}{\pi \left( \theta | \mathcal{D}_t \right)} \sum_{q_t \in \Omega(t)} \mu_{z | q_t, \theta, \mathcal{D}_t} \pi \left( q_t, \theta | \mathcal{D}_t \right).$$

The predictive covariance matrix of  $z|\theta, D_t$  can be obtained using the law of total variance  $\Sigma_{z|\theta,D_t} = \mathbb{E} \left( \Sigma_{z|qt,\theta,D_t} \right) + \text{Cov} \left( \mu_{z|qt,\theta,D_t} \right)$ , where the  $\mathbb{E}$  and Cov is computed over random variable  $q_t$ . Such variables are obtained as follows.

$$\mathbb{E} \left( \Sigma_{z|q_{t},\theta,D_{t}} \right) = \sum_{q_{t} \in \Omega(t)} \Sigma_{z|q_{t},\theta,D_{t}} \pi \left( q_{t}|\theta, D_{t} \right),$$

$$Cov \left( \mu_{z|q_{t},\theta,D_{t}} \right) = \sum_{q_{t} \in \Omega(t)} \left( \mu_{z|q_{t},\theta,D_{t}} - \mu_{z|\theta,D_{t}} \right)^{T} \pi \left( q_{t}|\theta, D_{t} \right),$$

$$\times \left( \mu_{z|q_{t},\theta,D_{t}} - \mu_{z|\theta,D_{t}} \right)^{T} \pi \left( q_{t}|\theta, D_{t} \right),$$

$$\times \Sigma_{z|\theta,D_{t}} = -\mu_{z|\theta,D_{t}} \mu_{z|\theta,D_{t}}^{T} + \frac{1}{\pi \left( \theta|D_{t} \right)},$$

$$\times \sum_{q_{t} \in \Omega(t)} \left( \Sigma_{z|q_{t},\theta,D_{t}} + \mu_{z|q_{t},\theta,D_{t}} \mu_{z|q_{t},\theta,D_{t}}^{T} \right) \pi \left( qt, \theta|D_{t} \right),$$

where the predictive mean and covariance of  $z|q_t, \theta, D_t$ are calculated using the Gaussian process regression as follows.

$$\begin{split} \mu_{z|q_{t},\theta,\mathcal{D}_{t}} &= \mu_{z|\theta,\mathcal{D}_{t-1}} \\ &+ \Sigma_{z|\theta,\mathcal{D}_{t-1}} H_{q_{t}}^{T} \Sigma_{\tilde{z}_{t}|\theta,\mathcal{D}_{t-1},q_{t}}^{-1} \left( \tilde{z}_{t} - \mu_{\tilde{z}_{t}|\theta,\mathcal{D}_{t-1},q_{t}} \right), \\ \Sigma_{z|q_{t},\theta,\mathcal{D}_{t}} &= \Sigma_{z|\theta,\mathcal{D}_{t-1}} \\ &- \Sigma_{z|\theta,\mathcal{D}_{t-1}} H_{q_{t}}^{T} \Sigma_{\tilde{z}_{t}|\theta,\mathcal{D}_{t-1},q_{t}}^{-1} H_{q_{t}} \Sigma_{z|\theta,\mathcal{D}_{t-1}}. \end{split}$$

Finally, the first and second moments of  $q_t | D_t$  are obtained as follows.

$$\begin{split} \mu_{q_t \mid \mathcal{D}_t} &= \sum_{q_t \in \Omega(t)} q_t \pi \left( q_t \mid \mathcal{D}_t \right), \\ \Sigma_{q_t \mid \mathcal{D}_t} &= \sum_{q_t \in \Omega(t)} \left( q_t - \mu_{q_t \mid \mathcal{D}_t} \right)^2 \pi \left( q_t \mid \mathcal{D}_t \right). \end{split}$$

The fully Bayesian field SLAM is now summarized as Algorithm 1 in the Appendix.

To demonstrate the usefulness of the proposed field SLAM, we apply it to two case studies under simulation and experimental environments in the following sections.

#### **III. SIMULATION STUDY**

In this section, we demonstrate the effectiveness of the proposed sequential Bayesian inference algorithm (*i.e.*, field SLAM) using a numerical experiment. We construct our simulation scenario as follows. The robot moves through a scalar field z that can be characterized by a GMRF model, and takes noisy measurements of its 2-D locations and the scalar field. We assume that we know the control signal that was sent to the robot and the initial prior distribution of the hyperparameter vector of the GMRF. In particular, we consider that a robot is moving in a discretized surveillance region S. The spatial sites in S consist of  $31 \times 31$  grid points, *i.e.*, n = 961,

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uniformly distributed over the surveillance region  $S_c := [-15, 15] \times [-15, 15]$ . The evolution of the robot location can be more detailed as follows.

$$q_{t+1} = Q(q_t + F_t u_t + v_t) = q_t + F_t u_t + w_t,$$
(9)

where  $Q: S_c \to S$  is the nearest neighbor rule quantizer that takes an input and returns a projected value on S.  $v_t$  is the process noise and  $w_t$  is the quantization error between the continuous and discretized states, *i.e.*,  $w_t :=$  $Q(q_t + F_t u_t + v_t) - (q_t + F_t u_t)$ . As the cardinality of Sincreases, we have that  $w_t \to v_t$ . A special case of (9) is that  $F_t u_t$  is controlled and  $w_t$  is chosen such that the next location  $q_{t+1}$  is on a grid point in S. In this case, we have  $v_t = w_t$ .

#### 3.1 GMRF configuration

In this simulation study, we realize the spatial field developed in [24], where a GMRF wrapped around in a torus structure. Thus the top edge (respectively, the left edge) and the bottom edge (respectively, the right edge) are neighbors to each other. The parameters of the model in [24] are selected as follows. The mean vector  $\mu_{\theta}$  is chosen to be zero, and the precision matrix  $Q_{\theta}$  is chosen with hyperparameters  $\alpha = 0.1$  equivalent to a bandwidth  $\ell = \sqrt{2}/\sqrt{\alpha} \approx 4.47$ , and  $\kappa = 50$  equivalent to  $\sigma_f^2 = 1/4\pi\alpha\kappa \approx 0.016$ . The prior distribution of the hyperparameter vector  $\theta$  is discrete with a support

$$\Theta = \{(\kappa, \alpha), (0.1\kappa, \alpha), (10\kappa, \alpha), (\kappa, 0.1\alpha), (\kappa, 10\alpha)\},\$$

along with the corresponding uniform probabilities  $\{0.2, 0.2, 0.2, 0.2, 0.2\}$ . The measurement noise variance in (1) is given by  $\sigma_{\epsilon} = 0.1$ .

A robot takes measurements at time  $t \in \{1, 2, \dots, 100\}$  with localization uncertainty. In Fig. 1d,e,f true, noisy, and probable sampling positions are shown in circles, stars, and dots, respectively, at time t = 100. In this simulation, the standard deviation of the noise in the observed sampling position is given by  $\sigma_e = 10$  in (2). The probable sampling positions that form support  $\Omega(t)$ , are selected within the confidence region of  $\Pr(\|q_t^{[i]} - \tilde{q}_t^{[i]}\| \le \sigma_e)$ .



Fig. 1. The prediction results of Cases 1, 2 and 3 at time t = 100 are shown in the first, second, third columns, respectively. The first, second, and third rows correspond to the prediction, prediction error variance, and squared empirical error fields between the prediction and true fields. True, noisy, and probable sampling positions are shown in circles, stars, and dots, respectively. The x and y axis represent 2-D localization, and colors represent the value of the desired quantity in the locations.

#### 3.2 Simulation results with accurate orientation

The results of the simultaneous localization and spatial prediction are summarized for three methods as follows.

- **Case 1.** Fig. 1a,d,g shows the prediction, prediction error variance, and squared (empirical) error fields, using exact sampling positions. With the true sampling positions, the best prediction quality is expected for this case.
- Case 2. Fig. 1b,e,h shows the resulting fields, by using sampled noisy positions. The results clearly illustrate that naively applying GMRF regression to noisy sampling positions can potentially distort prediction at a significant level. Fig. 1h shows that squared error of this case is considerably higher than that of Case 1.
- **Case 3.** Fig. 1c,f,i shows the resulting fields, by applying Algorithm 1. The resulting prediction quality is much improved as compared to Case 2 and is even comparable to the result for Case 1.

The true positions of the robot in the simulation for time  $t \in T_s := \{10, 11, \dots, 30\}$  are shown in Fig. 2 by red diamonds and lines. The estimated sampling positions of the robot  $\mathbb{E}(q_t|Dt)$  for  $t \in T_s$  are shown in blue dots with estimated confidence regions. Fig. 2 clearly shows that the proposed approach in this paper significantly reduces the localization uncertainty as compared



Fig. 2. Cases 1–3: The trajectories of true, predicted, and noisy sampling positions of the robot are shown by red diamonds, blue dots, and green stars for time t ∈ {10, 11, ..., 30}. The blue ellipsoids show the confidence regions of about 90% for the estimated sampling positions.

to the noise level of the sampled positions (denoted by green stars).

Table I shows the root mean squared errors (RMSE) in predictions of the scalar field and localizations using GMRF regression with true sampling positions (Case 1), GMRF regression with noisy sampling positions (Case 2) and the proposed approach with uncertain sampling positions (Case 3). This shows the effectiveness of our solution to Problem II.2.

#### 3.3 Simulation results with noisy orientation

In this section, we show how to relax the condition that  $\psi_t$  is not known and  $\psi_t$  needs to be computed from the noisy turn rate  $\eta_t$  using the EKF. We extend the evolution of the robot location described in (9) to include the orientation state as follows. For the sake of simplicity, we will describe the algorithm for one robot.

Define  $\mathbf{x}_t \in \mathbb{R}^3$  as the state vector that includes position and orientation of the robot, *i.e.*,  $x_t = [q_t^{[1]}, q_t^{[2]}, \psi_t]^T$ . Therefore, we have the state transition equation of the mobile robot:

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \Delta t \begin{bmatrix} u_t \cos \psi_t \\ u_t \sin \psi_t \\ \eta_t \end{bmatrix} + \xi_t$$

$$= f \left( \mathbf{x}_t, u_t, \eta_t \right) + \xi_t,$$
(10)

where  $\xi_t \sim \mathcal{N}(0, \Sigma_{\xi_t})$  is the system process noise. Notice that  $\xi_t \in \mathbb{R}^3$  includes  $w_t$  in (9) and the noise on the angular rate. The robot takes measurement of its own spatial location, *i.e.*,

$$\tilde{q}_t = M\mathbf{x}_t + e_t,\tag{11}$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

We consider two localization schemes as follows.

Table I. The RMSE	of	predicted	random
field and localization			

	RMSE					
	Predicted field	Localization				
Case 1	0.0862	0.00*				
Case 2	0.1133	6.74				
Case 3	0.1021	3.63				

\*Since Case 1 uses exact sampling positions, the localization error is zero.

	Method	Averaged RMSE
Case 4	EKF	12.3
Case 5	GP-EKF	9.9

Table II. Comparison between GP-EKF and EKF localization

- Case 4. EKF recursively predicts the location for the next iteration using (10), then corrects the noisy measurement  $\tilde{q}_t$  based on the updated covariance matrix. Notice that in Case 4, the EKF does not utilize the field measurement.
- Case 5. We apply our proposed method as a post-processing step that utilizes the results from the EKF in Case 4, *i.e.*,  $\tilde{q}_t$  and  $\psi_t$  are the estimated states from the EKF.

We present a Monte Carlo simulation result with 50 iterations to compare our proposed method with the EKF. The averaged RMSEs are shown in Table II. The EKF and our proposed methods are indicated as "EKF" and "GP-EKF", respectively. Table II clearly indicates that our method outperforms the EKF by 19.5%.

In this example study, the fixed running time using MATLAB on a PC (3.2 GHz Intel i7 Processor, Intel Corporation, Santa Clara, CA, USA) is about 100 seconds for each iteration of time, which is fast enough for a real world implementation.

#### **IV. EXPERIMENTAL STUDY**

In this section, we present experiment results when our scheme is applied to vision-based localization. We built a two-wheeled mobile robot equipped with a panoramic vision camera, a micro-controller, and two wireless routers as shown in the right-side of Fig. 3. The micro-controller (Arduino MEGA board, Open Source Hardware platform, Italy) runs a proportional-integral-derivative (PID) controller. One wireless router (TP-Link TL-WR703N 150M, TP-LINK Inc., San Dimas, CA, USA) is used for streaming the video recorded from the web-cam (Logitech HD webcam C310, Logitech, Newark, CA, USA) equipped with a 360° panoramic lens (Kogeto Panoramic Dot Optic Lens, Kogeto, New York, NY, USA) and another router is used for receiving the control commands remotely via the Internet.

The robot takes command inputs and a low-level PID controller programmed in a micro-controller tracks the inputs so that the mobility of the robot can be described by

$$q_{t+1} = q_t + \Delta t \begin{bmatrix} u_t \cos \psi_t \\ u_t \sin \psi_t \end{bmatrix},$$
(12)

where  $\Delta t$  is the sampling time.  $q_t, \psi_t$ , and  $u_t$  are the robot position, the robot orientation, and the command input, respectively. The orientation  $\psi_t$  is re-calculated by the given turn rates so that (12) is updated online.

Fig. 3 shows the mobile robot in the testing environment. The mobile robot is controlled remotely by a user while the panoramic vision of the surrounding scene is recorded. Another overhead web camera (Logitech HD webcam C910, Logitech, Newark, CA, USA) is mounted on the ceiling to measure the ground truth positions of the robot for evaluating the proposed approach. Two PCs are used in the experiment, *i.e.*, one for sending command inputs to the robot and the other one for



Fig. 3. Left: The testing environment. Right: The mobile robot equipped with a panoramic vision camera.

processing the images from the overhead web camera for tracking robot's trajectory for later evaluation, and recording the scene streamed from the panoramic vision camera installed on the robot. In summary, there are three data sets collected from the experiment, all sampled at 0.5 Hz. The command inputs, turn rates, and sampling times  $\{u_t, \eta_t, \Delta_t\}$  are sampled. The scene recorded by the panoramic vision web-cam is streamed on-line. The positions of the mobile robot are tracked by the overhead camera.

#### 4.1 Informative scalar field

In this section, we select an informative scalar field from the vision data to be useful in localization. For each sampling time, the frame (doughnut shape image in Fig. 4a) from the video captured by the panoramic vision camera is unwrapped into a panoramic image. The height is the thickness of the doughnut shape image (Fig. 4a) and the width is  $2\pi \times r_{outer}$ , where  $r_{outer}$  is its outer radius. Each panoramic image is converted into a gray scale and resized as a square picture as shown in Fig. 4b. To find an informative feature from the vision data, a fast Fourier Transform (FFT) is applied to the square images. For an image of size  $M \times M$  captured at sampling time t, the two-dimensional FFT is given by

$$F^{[l]}(\mathfrak{u},\mathfrak{v}) = \sum_{a=0}^{M-1} \sum_{b=0}^{M-1} f^{[l]}(a,b) \exp\left(-j2\pi\left(\mathfrak{u}\frac{a}{M} + \mathfrak{v}\frac{b}{M}\right)\right)$$
$$\forall \mathfrak{u},\mathfrak{v} \in \{0,2,\cdots,M-1\},$$
(13)

where  $f^{[t]}(a, b)$  is the gray scale square image in the spatial coordinates, **u** and **v** are coordinates in the frequency domain, and *j* is the imaginary unit. For example, the two-dimensional magnitude plot from one image is shown in Fig. 4c. Note that energy waves mainly distribute along the horizontal axis due to the dominant

vertical patterns occurring in the surrounding vision images. Note that only the phase of the FFT is effected by the direction of the robot while the magnitude is robust with respect to the heading angle. Therefore, we utilize the magnitude of the FFT of the features distributed in the horizontal direction from (13) as follows.

$$m^{[t]}(\mathfrak{u}) = |F^{[t]}(\mathfrak{u}, 0)|,$$

where  $|\cdot|$  denotes the magnitude of a complex number. To extract a good feature value, we fit  $m^{[t]}(\mathfrak{u})$  to an exponential function:

$$\ln \hat{m}^{[t]}(\mathfrak{u}, a_1(t), a_0(t)) = a_1(t)\mathfrak{u} + \ln a_0(t),$$
$$\forall \mathfrak{u} \in \{0, \cdots, M-1\}.$$

The noisy scalar field observation by the robot at t is now considered to be

$$\{a_1^{\star}(t), a_2^{\star}(t)\} = \underset{a_1(t), a_2(t)}{\operatorname{argmin}} \sum_{u=0}^{M-1} \left\{ \ln \hat{m}^{[t]} - \ln m^{[t]} \right\}$$
$$\tilde{z}(t) := a_1^{\star}(t).$$

The standardized measurements  $\{\tilde{z}(t)\}\$  are plotted in Fig. 4d.

In this section, we briefly explained how the informative scalar field was selected for vision-based localization. However, the feature selection for this application is another rich research topic in robotics and machine learning communities [34], which is beyond the scope of this paper.

#### 4.2 GMRF modeling

In this section, we show how to model z(t) via a GMRF model. The discretized surveillance region S consists of 41×61 grid points (instead of 31×31 as in the



Fig. 4. Feature extraction process: (a) The original doughnut shape image. (b) The resized panoramic image. (c) The zoomed-in frequency-domain image. (d) The sampled (standardized) scalar field of interest over time iteration.

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Fig. 5. The probabilities distribution of the hyperparameters vector  $\theta$  at (a) t = 1, (b) t = 3 and (c) t = 40.



Fig. 6. The top-view of the prediction error variance plotted with the real robot's trajectory in dashed white line.

simulation study) to be consistent with the frame ratio of the overhead camera.

The maximum likelihood estimation (MLE) [20] of the hyperparameter vector is found, which yields  $\alpha =$  0.0661 (equivalent to a bandwidth  $\ell =$  5.5), and  $\kappa =$  1.672 (equivalent to  $\sigma_f^2 = 0.72$ ).

To test the data-driven adaptability of our fully Bayesian SLAM, we construct a prior distribution (as if we don't have the MLE) centered around the MLE such as

$$\Theta = \{ (\kappa, \alpha), (0.1\kappa, \alpha), (10\kappa, \alpha), (\kappa, 0.1\alpha), (\kappa, 10\alpha) \},\$$

along with the corresponding uniform probabilities  $\{0.2, 0.2, 0.2, 0.2, 0.2\}$ . We then investigate the posterior distribution of  $\theta$  over the support set  $\Theta$ . Note that  $\Theta$  contains the MLE ( $\kappa$ ,  $\alpha$ ) hoping that our scheme converges to this MLE. The measurement noise level is set by  $\sigma_{\epsilon} = 0.1$ . The running time (using the same PC as in the simulation



Fig. 7. The 3D view of the predicted scalar field and measurement of the robot at each sampling point are plotted in gray surface and red line, respectively.



Fig. 8. The ground truth, predicted path and noisy observation are plotted in red diamonds, blue dots and green stars, respectively.

case) is about 1000 seconds for each iteration, which can be reduced when the structure of the loop in Algorithm 1 is exploited for parallelized computation.

#### 4.3 Experimental results

The mobile robot moves through the testing environment and takes measurements for  $t \in \mathcal{T}_e$  :=  $\{1, \dots, 40\}$ . The updated discrete probability distribution of the hyperparameter vector  $\theta$  in (8) at the sampling times t = 1, t = 3 and t = 40 are shown in Fig. 5. Fig. 5 clearly demonstrates the convergence of the posterior distribution of  $\theta$  by the peak at the MLE ( $\kappa, \alpha$ ).

The top-view of the prediction error variance is depicted in Fig. 6 with true positions of the robot plotted in a dashed, white line. The 3D view of the predicted field is plotted in Fig. 7 along with the interpolated measurements from the robot over all sampling points. In contrast to the simulation case, the robot mobility model may have large modeling error due to the difference between the model and real robot dynamics. The RMSE of predicted robot locations when only using open-loop kinematics is 0.3393 (m) over the length of a 9 (m) trajectory. The estimated sampling positions of the robot for all  $t \in \mathcal{T}_e$  are shown in blue dots in Fig. 8 along with the ground truth and noisy measured positions in red diamonds and green stars, respectively. Our proposed approach reduces the RMS error to 0.1332 (m) with 61% improvement from the open-loop prediction.

# V. CONCLUSION

In this paper, we provide an approximate Bayesian solution to the problem of simultaneous localization and spatial prediction (field SLAM), taking into account kinematics of robots and uncertainties in the precision matrix, the sampling positions, and the measurements of a GMRF in a fully Bayesian manner. In contrast to [24], the kinematics of the robotic vehicles are integrated into the inference algorithm. The simulation results show that the proposed approach estimates the sampling positions and predicts the spatial field along with their prediction error variances successfully, in a fixed computational time. The feasibility of the proposed approach is tested in a real world experiment. The experiment results indicate the adaptability of the approach to handle the noise from location measurement, scalar field observations, and the uncertainty of the kinematics model in a data-driven fashion.

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#### **VI. APPENDIX**

The proposed fully Bayesian SLAM algorithm is summarized in Algorithm 1.

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Input:	(1) prior distribution of $\theta \in \Theta$ ,
	(2) input matrix $F_t$ and control input $u_t$ .
	(3) prior distribution of noise $\epsilon$ , and disturbance $w_t$ .
Output:	(1) predictive mean $\mu_{z \mathcal{D}_t}$ and predictive covariance matrix $\Sigma_{z \mathcal{D}_t}$

At time  $t \in \mathbb{Z}_{>0}$ , do:

1: obtain new observations  $\{\tilde{z}_t, \tilde{q}_t\}$  collected at current locations  $q_t$ .

2: approximate the predictive distribution  $q_t | \mathcal{D}_{t-1}$ 

$$\pi \left( q_t | \mathcal{D}_{t-1} \right) = \mathcal{N} \left( \mu_{q_{t-1} | \mathcal{D}_{t-1}} + F_{t-1} u_{t-1}, \quad \Sigma_{q_{t-1} | \mathcal{D}_{t-1}} + \Sigma_{w_{t-1}} \right).$$

- 3: build support  $\Omega(t)$  from  $\pi(q_t | \mathcal{D}_{t-1})$ .
- 4: for  $q_t \in \hat{\Omega}(t)$  do
- 5: find the map  $H_{q_t}$  from  $q_t$  to spacial sites S.
- 6: for  $\theta \in \Theta$  do
- 7: The predictive statistics of the observed data  $\tilde{z}_t$  at the possible sampling position  $q_t$  are computed as follow

$$\mu_{\tilde{z}_t|\theta,\mathcal{D}_{t-1},q_t} = H_{q_t}\mu_{z|\theta,\mathcal{D}_{t-1}}, \quad \Sigma_{\tilde{z}_t|\theta,\mathcal{D}_{t-1},q_t} = \Sigma_{\epsilon_t} + H_{q_t}\Sigma_{z|\theta,\mathcal{D}_{t-1}}H_{q_t}^T.$$

8: approximate the distribution of  $q_t, \theta | \mathcal{D}_t$ 

$$\pi \left( q_t, \theta | \mathcal{D}_t \right) \propto \pi \left( \tilde{q}_t | q_t \right) \pi \left( \theta | D_{t-1} \right) \pi \left( q_t | D_{t-1} \right) \pi \left( \tilde{z}_t | q_t, \theta, \mathcal{D}_{t-1} \right)$$

9: using Gaussian process regression

$$\mu_{z|q_{t},\theta,\mathcal{D}_{t}} = \mu_{z|\theta,\mathcal{D}_{t-1}} + \Sigma_{z|\theta,\mathcal{D}_{t-1}} H_{q_{t}}^{T} \Sigma_{\tilde{z}_{t}|\theta,\mathcal{D}_{t-1},q_{t}}^{-1} (\tilde{z}_{t} - \mu_{\tilde{z}_{t}|\theta,\mathcal{D}_{t-1},q_{t}}),$$
$$\Sigma_{z|q_{t},\theta,\mathcal{D}_{t}} = \Sigma_{z|\theta,\mathcal{D}_{t-1}} - \Sigma_{z|\theta,\mathcal{D}_{t-1}} H_{q_{t}}^{T} \Sigma_{\tilde{z}_{t}|\theta,\mathcal{D}_{t-1},q_{t}}^{-1} H_{q_{t}} \Sigma_{z|\theta,\mathcal{D}_{t-1}}$$

10: end for

- 11: end for
- 12: marginalize out uncertainties

$$\begin{aligned} \pi\left(q_{t}|\mathcal{D}_{t}\right) &= \sum_{\theta\in\Theta} \pi\left(q_{t},\theta|\mathcal{D}_{t}\right), \quad \pi\left(\theta|\mathcal{D}_{t}\right) = \sum_{q_{t}\in\Omega(t)} \pi\left(q_{t},\theta|\mathcal{D}_{t}\right), \\ \mu_{q_{t}|\mathcal{D}_{t}} &= \sum_{q_{t}\in\Omega(t)} q_{t}\pi\left(q_{t}|\mathcal{D}_{t}\right), \quad \Sigma_{q_{t}|\mathcal{D}_{t}} = \sum_{q_{t}\in\Omega(t)} q_{t}^{2}\pi\left(q_{t}|\mathcal{D}_{t}\right) - \mu_{q_{t}|\mathcal{D}_{t}}^{2} \\ \mu_{z|\theta,\mathcal{D}_{t}} &= \frac{1}{\pi\left(\theta|\mathcal{D}_{t}\right)} \sum_{q_{t}\in\Omega(t)} \mu_{z|q_{t},\theta,\mathcal{D}_{t}}\pi\left(q_{t},\theta|\mathcal{D}_{t}\right), \\ \Sigma_{z|\theta,\mathcal{D}_{t}} &= \frac{1}{\pi\left(\theta|\mathcal{D}_{t}\right)} \sum_{q_{t}\in\Omega(t)} \left(\Sigma_{z|q_{t},\theta,\mathcal{D}_{t}} + \mu_{z|q_{t},\theta,\mathcal{D}_{t}}\mu_{z|q_{t},\theta,\mathcal{D}_{t}}\right)\pi\left(q_{t},\theta|\mathcal{D}_{t}\right) - \mu_{z|\theta,\mathcal{D}_{t}}\mu_{z|\theta,\mathcal{D}_{t}}^{T} \\ \mu_{z|\mathcal{D}_{t}} &= \sum_{\theta\in\Theta} \mu_{z|\theta,\mathcal{D}_{t}}\pi\left(\theta|\mathcal{D}_{t}\right), \quad \Sigma_{z|\mathcal{D}_{t}} = \sum_{\theta\in\Theta} \left(\Sigma_{z|\theta,\mathcal{D}_{t}} + \mu_{z|\theta,\mathcal{D}_{t}}\mu_{z|\theta,\mathcal{D}_{t}}^{T}\right)\pi\left(\theta|\mathcal{D}_{t}\right) - \mu_{z|\mathcal{D}_{t}}\mu_{z|\mathcal{D}_{t}}^{T}, \end{aligned}$$



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